Name _____Course Days/Start Time _____

Mini-Project 1: The Library of Functions and Piecewise-Defined Functions

Part A: The Library of Functions

In your previous math class, you learned to graph equations containing two variables by finding and plotting points. In this class, we continue this skill by learning to graph functions that are often used in algebra. Collectively, we call these functions the library of functions.

To graph a function from the library of functions, follow these steps:

1. Assume a value for x, then solve for the value of y. (Remember that $f(x)$ is the same thing as y.)
2. Express these values of x and y as an ordered pair.
3. Repeat Steps 1 and 2 until you have enough points.
4. Determine the scale of the x-axis and the y-axis.
5. Plot the ordered pairs, then connect the points to graph the equation.

We will practice each of these steps, starting with Steps 1, 2 and 3.

Steps 1, 2, and 3

In class, we learned the values of x we should use to graph each of the functions in the library of functions. Find those values in your lecture notes, then answer these questions:

1. What values of x should you use if you are graphing the Constant Function? (Circle One.)

$\boxed{A}$ -2, -1, 0, 1, and 2 $\boxed{B}$ 0, 1, 4, 9, and 16 $\boxed{C}$ -8, -1, 0, 1, and 8 $\boxed{D}$ $-4, -2, -1, -\frac{1}{2}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{2}$, 1,2 and 4

2. What values of x should you use if you are graphing the Square Root Function? (Circle One.)

$\boxed{A}$ -2, -1, 0, 1, and 2 $\boxed{B}$ 0, 1, 4, 9, and 16 $\boxed{C}$ -8, -1, 0, 1, and 8 $\boxed{D}$ $-4, -2, -1, -\frac{1}{2}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{2}$, 1,2 and 4

3. What values of x should you use if you are graphing the Cube Function? (Circle One.)

$\boxed{A}$ -2, -1, 0, 1, and 2 $\boxed{B}$ 0, 1, 4, 9, and 16 $\boxed{C}$ -8, -1, 0, 1, and 8 $\boxed{D}$ $-4, -2, -1, -\frac{1}{2}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{2}$, 1,2 and 4

4. What values of x should you use if you are graphing the Reciprocal Function? (Circle One.)

$\boxed{A}$ -2, -1, 0, 1, and 2 $\boxed{B}$ 0, 1, 4, 9, and 16 $\boxed{C}$ -8, -1, 0, 1, and 8 $\boxed{D}$ $-4, -2, -1, -\frac{1}{2}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{2}$, 1,2 and 4

Once the values of x have been determined, plug each one in, one at a time, to find the corresponding values of y. Then express each x- and y-value as an ordered pair.

For each function listed below, use the x-values to find the corresponding y-values, then express them as an ordered pair.

5. The Square Function

$f(x) = x^2$

x	y	(x, y)
-2		
-1		
0		
1		
2	4	

6. The Identity Function

$f(x) = x$

x	y	(x, y)
-2		
-1		
0		$(0,0)$
1		
2		

7. The Cube Root Function

$f(x) = \sqrt[3]{x}$

x	y	(x, y)
-8		
-1		
0		
1		
8		

For the reciprocal function, each x-value is the reciprocal of its corresponding y-value and vice versa. For example, if $x = \frac{1}{4}$, then $y = \frac{4}{1}$ or simply 4. If a value is negative, then its reciprocal is negative. For example, the reciprocal of -2 is $-\frac{1}{2}$.

8. The Reciprocal Function

$f(x) = \frac{1}{x}$

x	y	(x, y)
-4		
-2	$-\frac{1}{2}$	
-1		
$-\frac{1}{2}$		
$-\frac{1}{4}$		
$\frac{1}{4}$	4	
$\frac{1}{2}$		
1		
2		
4		

Step 4

To determine the scale of the x-axis and the y-axis, identify the smallest value and the largest value in your table of points to be plotted. Start your axis numbering with a value that is equal to or "a little bit" less than the smallest value, and stop at a value that is equal to or "a little bit" more than the largest value. Use "convenient" numbers, like multiples of 5 or 10.

For example, if your smallest value is -13 and your largest value is 2, you should start your axis at -15 and stop your axis at 5. You would label increments of 5, so you would write -15, -10, and -5 left of/below the origin, and 5 right of/above of the origin. In this case, the origin would <u>not</u> be in the center of the graph.

For each table of values listed below, select an appropriate x- and y-scale.

x	y
0	0
1	1
4	2
9	3
16	4

9. An appropriate x- and y-scale would be… (Circle One.)

$\boxed{A}$ From 0 to 16 in increments of 1

$\boxed{B}$ From -20 to 20 in increments of 10

$\boxed{C}$ From 0 to 15 in increments of 5

$\boxed{D}$ From 0 to 20 in increments of 5

x	y
-2	-8
-1	-1
0	0
1	1
2	8

10. An appropriate x- and y-scale would be… (Circle One.)

$\boxed{A}$ From -5 to 5 in increments of 5

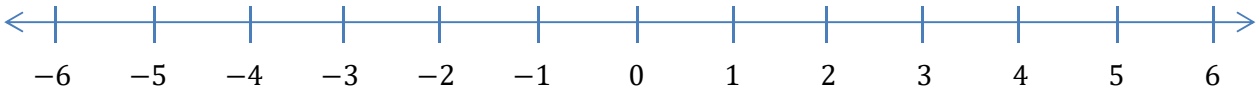
$\boxed{B}$ From -10 to 10 in increments of 5

$\boxed{C}$ From -2 to 8 in increments of 5

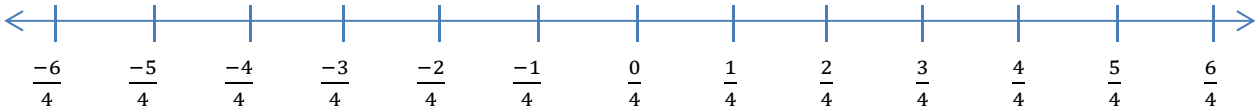
$\boxed{D}$ From -2 to 2 in increments of 1

When making the x-axis and the y-axis for the reciprocal function, you must be careful because fractions are involved. To learn how to create a number line with fractions, follow these steps:

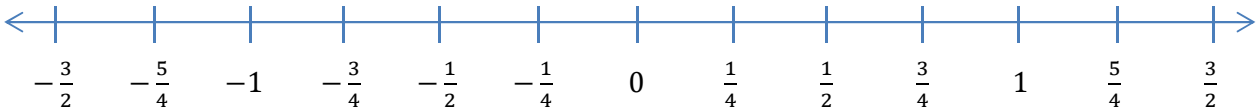
First, draw a number line like you would normally see in a textbook: all integers with zero in the middle.



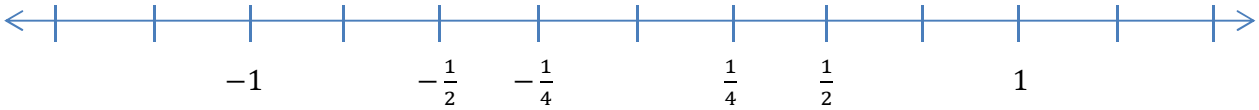
Next, divide each value by the largest denominator of all the fractions you need to plot. For the reciprocal function, that value is 4. This will ensure your fractions are in the right place, in the right order.



Finally, reduce each fraction to lowest terms.



Note that for the reciprocal function, not all fraction values must be labeled. For example, you can keep just the values shown below:



Note also that there are four spaces between 0 and 1. So there should be four spaces between any adjacent integers. Use this idea to find where to place the numbers -4, -2, 2, and 4.

Step 5

It is a good idea to memorize the shapes of all eight functions in the library of functions. This will help you to graph them correctly when it comes time to plot your points and connect them. Consider the following:

The Square Function

The Constant Function

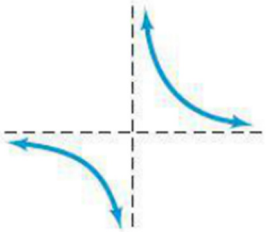
The Square Root Function

The Identity Function

The Cube Function

The Cube Root Function

The Reciprocal Function



The Absolute Value Function

The Constant, Identity, and Absolute Value functions should be drawn using a straightedge.
The reciprocal function has two asymptotes: $y = 0$ (the x-axis) and $x = 0$ (the y-axis). Graph them!

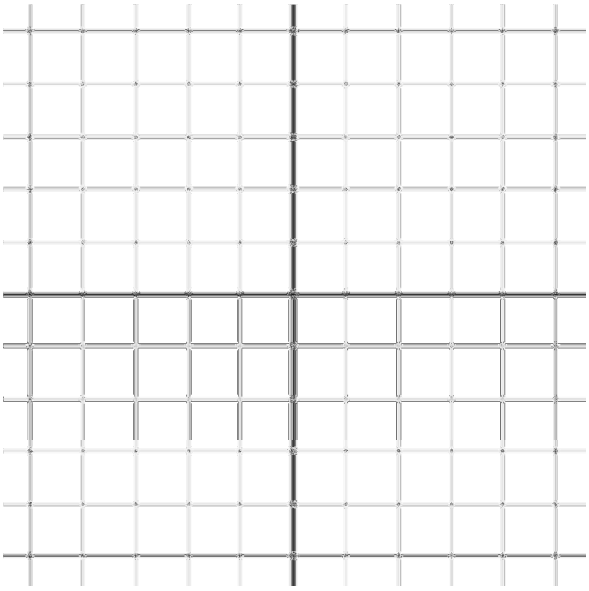
You are now ready to graph each function in the library of functions. There are eight tables and eight graphs. The first thing you should do is figure out which function goes with which graph based on the part of the graph shown and the information provided. Once you've done that, create all eight graphs. Fill in any missing information, including the function name, the function equation, and the table of points. **Be sure to use a straightedge where appropriate!**

11. Function Name:

The Identity Function

Function Equation:

x	y	(x, y)

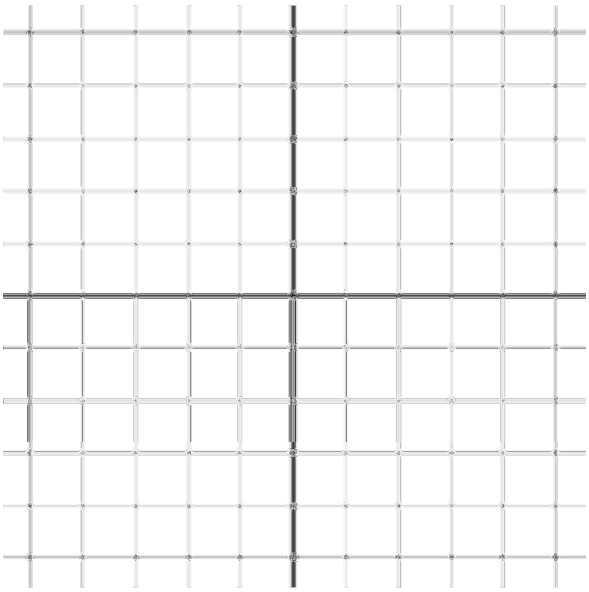


12. Function Name:

Function Equation:

$$\boldsymbol{f(x) = 4}$$

x	y	(x, y)



13. Function Name:

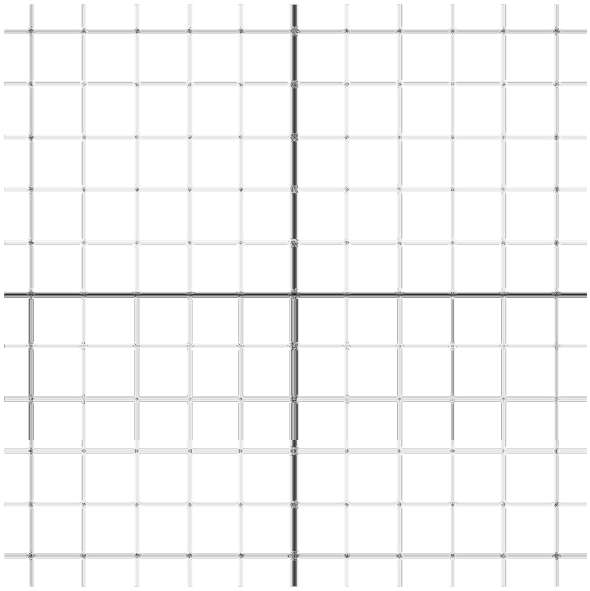
_____

Function Equation:

$$\boldsymbol{f(x) = x^2}$$

_____

x	y	(x, y)



14. Function Name:

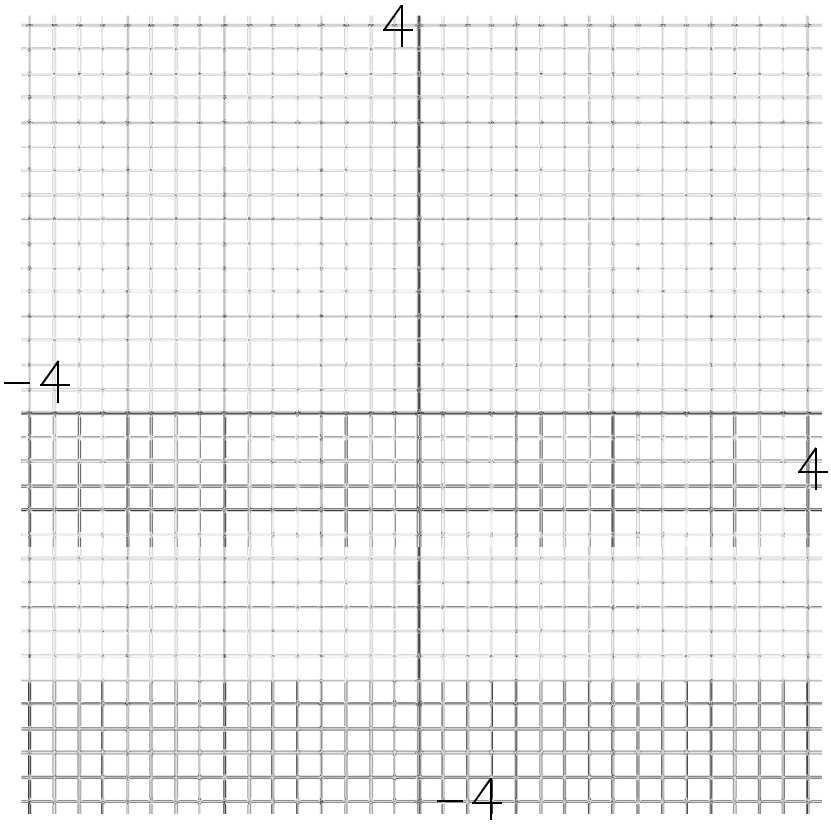
_____

Function Equation:

_____

Graph both asymptotes using dashed lines!

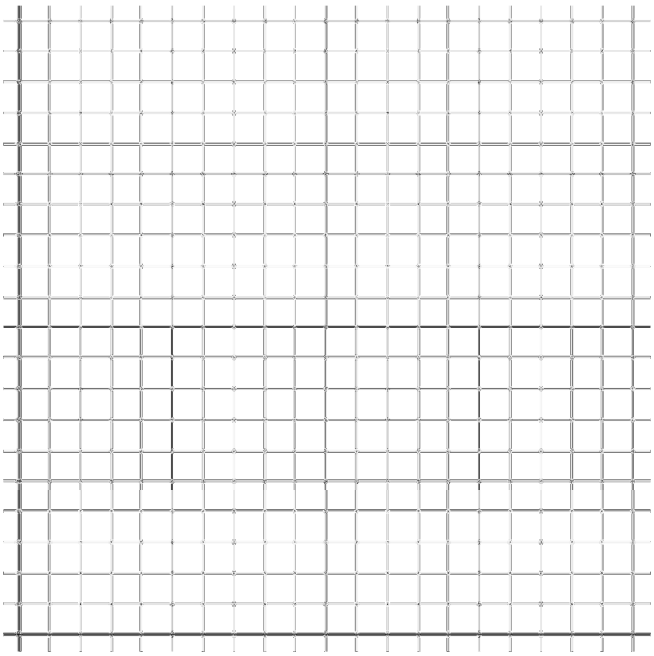
x	y	(x, y)



15. Function Name:

Function Equation:

x	y	(x, y)
16		

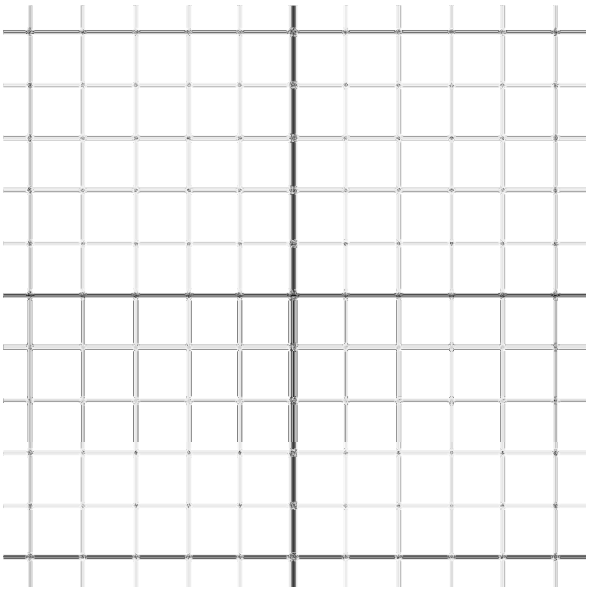


16. Function Name:

The Absolute Value Function

Function Equation:

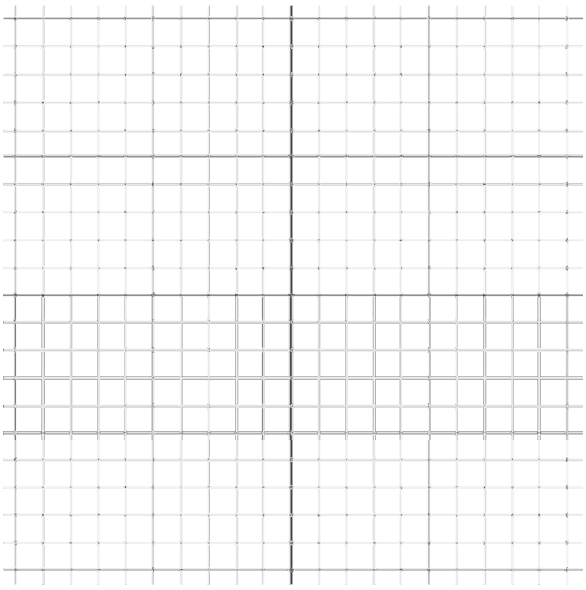
x	y	(x, y)
-2	2	
		$(2, 2)$



17. Function Name:

Function Equation:

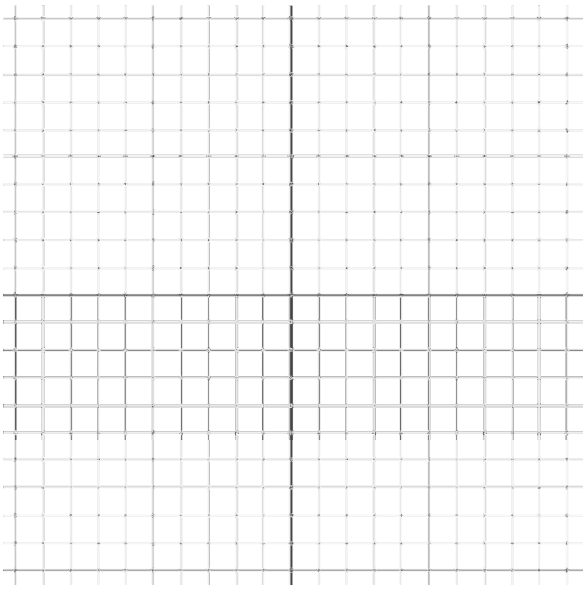
x	y	(x, y)
-8	-2	



18. Function Name:

Function Equation:

x	y	(x, y)
		$(2, 8)$



Checklist for a Perfect Graph

For each of the graphs you created, now go back and make sure you've satisfied the following requirements:

- The x-axis is drawn and labeled with an "x".
- The y-axis is drawn and labeled with a "y".
- The values on the x- and y-axis are stated.
    - For example, if you chose "From 0 to 20 in increments of 5," then you should write the 5, 10, 15, and 20 on the x-axis and the y-axis.
- The values on the x-axis and y-axis are consistent.
    - For example, if there are four spaces between 0 and 1, then there should be four spaces between 1 and 2, and so on.
- Correct points are plotted, and they are connected with the correct shape.
- The graph continues to the edge of the grid; arrows indicate that the graph continues beyond it.
- Straight lines and line segments are drawn with a straightedge.

Part B: Piecewise-Defined Functions

A piecewise-defined function is two or more functions combined into one. Each of the "pieces" is a function, but is used only for certain values of the input variable x. For example, the three functions $f(x) = x + 4$, $f(x) = 2$, and $f(x) = -x + 4$ can all be combined into the single piecewise-defined function shown below.

$$f(x) = \begin{cases} x + 4 & if\ x < 0 \\ 2 & if\ x = 0 \\ -x + 4 & if\ x > 0 \end{cases}$$

In class, you learned how to graph piecewise-defined functions. Here are the steps:

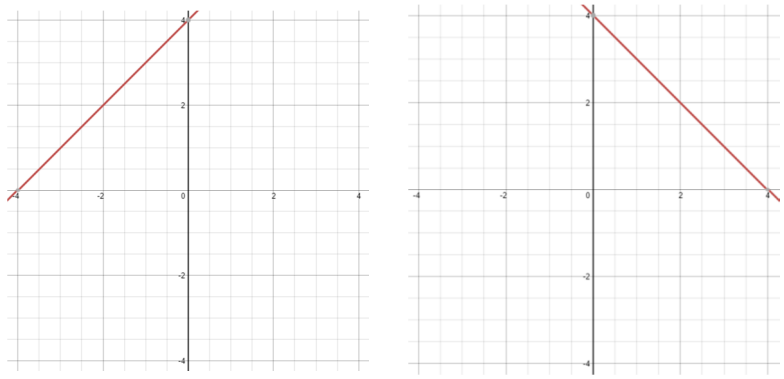
1. Create one graph for each piece whose "if" condition specifies more than just a single value for x. (In the example above, that's the top and bottom pieces.)
2. Cut each graph vertically using the x-values specified in the "if" condition. Keep only the relevant parts, and paste them together to begin creating your final graph. Be sure to line up the x-axis.
3. Add a point to your final graph for each piece whose "if" condition specifies exactly one value for x. (In the example above, that's the middle piece which says that when $x = 0$, $f(x) = 2$. Hence, you would add the point $(0,2)$ to your final graph.)
4. Place open and closed dots on the cut lines for each piece. Use a closed dot when the symbol $\leq$, $\geq$, or $=$ is next to an x-value in the "if" condition. Use an open dot when the symbol $<$, $>$, or $\neq$ is next to an x-value in the "if" condition.
5. Follow the "Checklist for a Perfect Graph" from Part A of this mini-project to put the finishing touches on your graph.

For example, to graph the piecewise-defined function,…

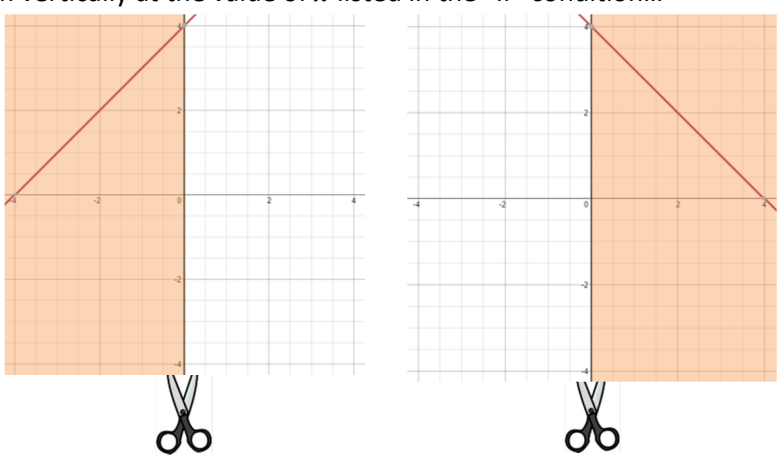
$$f(x) = \begin{cases} x+4 & if\ x<0 \\ 2 & if\ x=0 \\ -x+4 & if\ x>0 \end{cases}$$

…start by graphing…

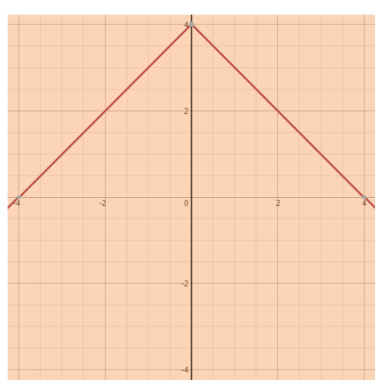
$f(x) = x + 4,$ and $f(x) = -x + 4$



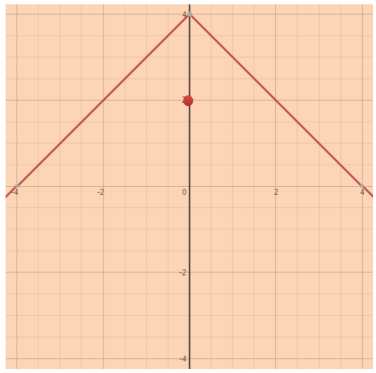
…then cut them vertically at the value of x listed in the "if" condition…



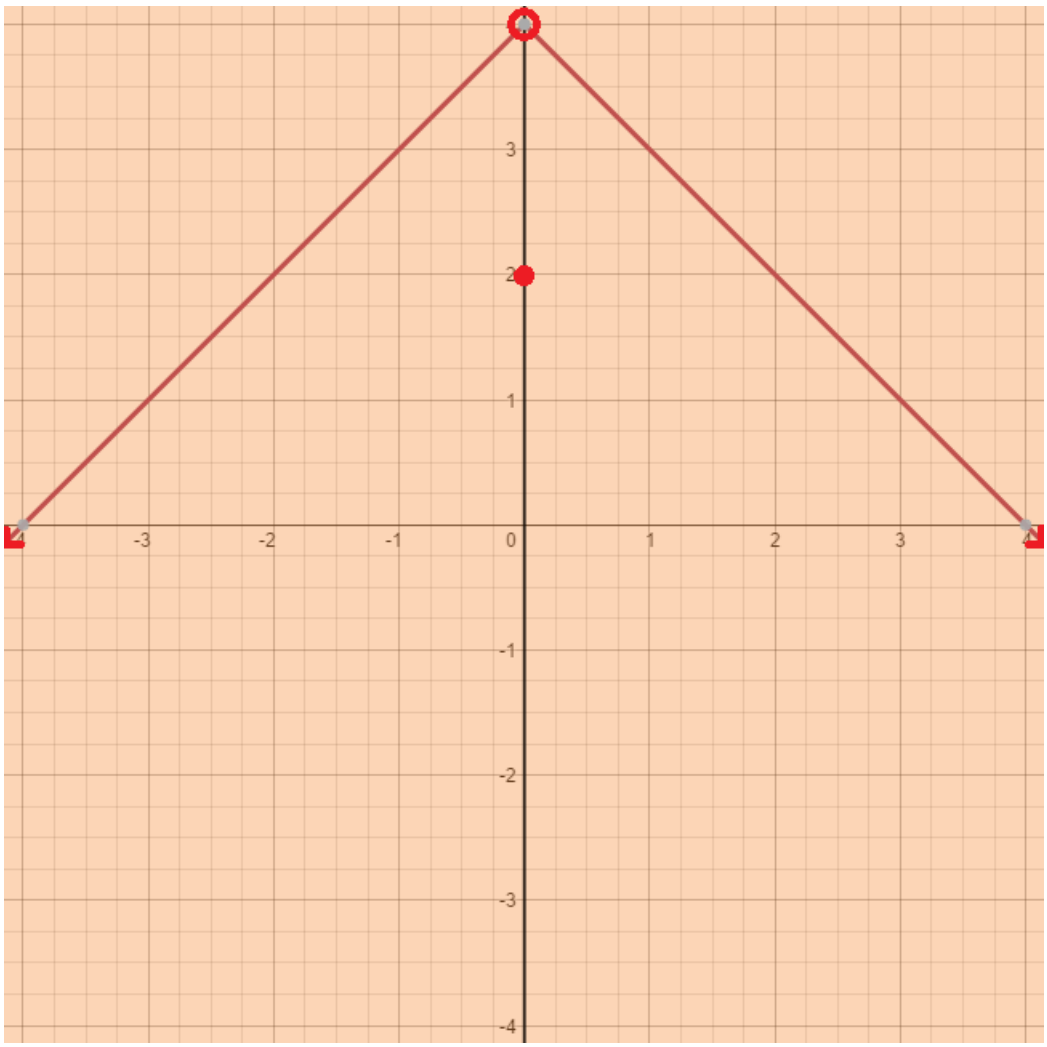
…to get two shaded graph pieces. Paste them together (line up the x-axis in each piece) to create the graph of the piecewise-defined function.



Add the point $(0,2)$ to this graph because the middle function states that when $x = 0$, $f(\mathrm{x}) = 2$.



As a final touch, put open dots and closed dots on your graph at the "cut" lines. The final graph is shown below.



Domain: $(-\infty, \infty)$ Range: $(-\infty, 4)$ Continuous?: No

Notice that the first piece has an arrow where it hits the edge of the graph and an open dot at the cut line. The middle piece produces just a single closed dot because it is only used for exactly one value of x. That value, $x = 0$, triggers the use of the middle function $f(x) = 2$. And for any input, that function produces an output of 2. $(y = 2)$ Hence, the graph shows the single closed dot $(0,2)$. The third piece starts with an open dot at the cut line and has an arrow where it hits the edge of the graph.

Notice also that the open dots from pieces 1 and 3 are exactly the same point, and are therefore drawn one atop the other.

Expert Tip: When a closed dot and an open dot are in the same place, the closed dot "fills" the open dot so that only the closed dot is visible.

Exercises

1. For the piecewise-defined function shown below, create its graph by cutting and pasting the graphs provided at the end of this packet. (Be sure to follow all five steps described on page 11.) Then answer the questions that follow.

$$f(x) = \begin{cases} -3x + 5 & if\ 0 < x < 1 \\ 2 & if\ x = 1 \\ 3x - 1 & if\ x > 1 \end{cases}$$

Using interval notation, what is the domain of $f(x)$? _____

Using interval notation, what is the range of $f(x)$? _____

Is $f(x)$ continuous on its domain? _____

2. For the piecewise-defined function shown below, create its graph by cutting and pasting the graph provided at the end of this packet. (Be sure to follow all five steps described on page 11.) Then answer the questions that follow.

$$g(x) = \begin{cases} x & if\ x \neq 0 \\ -2 & if\ x = 0 \end{cases}$$

Using interval notation, what is the domain of $g(x)$? _____

Using interval notation, what is the range of $g(x)$? _____

Is $g(x)$ continuous on its domain? _____

3. Consider the piecewise-defined function shown below. At the end of this packet, find the blank grids for each of the three pieces. Create those graphs yourself by hand. Then create the graph of the piecewise-defined function below by cutting and pasting the graphs you created. Answer the questions that follow.

$$h(x) = \begin{cases} 3 & if -4 < x < -1 \\ |x| & if -1 \leq x \leq 4 \\ \sqrt{x} & if \quad 4 < x \leq 16 \end{cases}$$

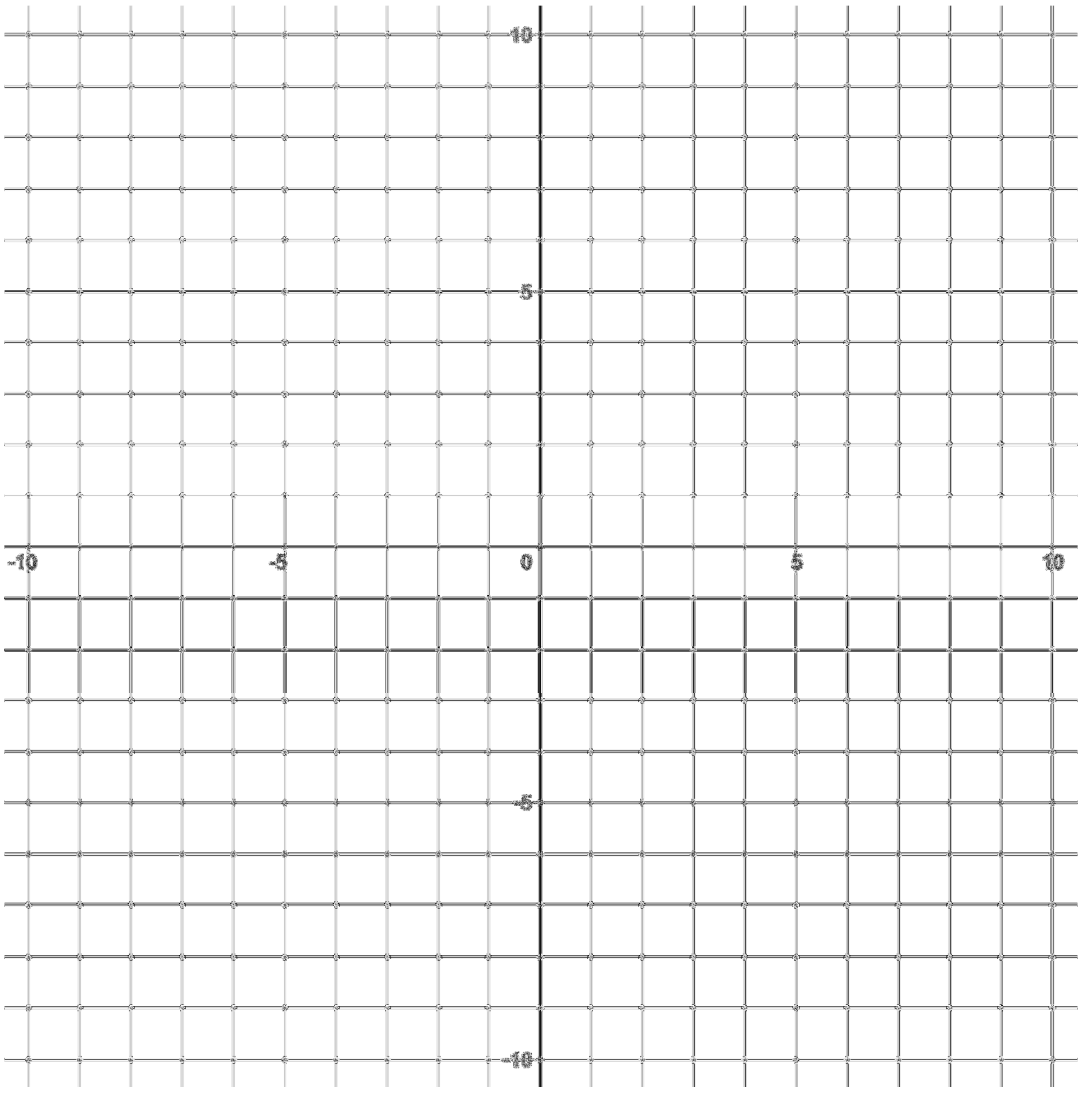
Using interval notation, what is the domain of $h(x)$? _____

Using interval notation, what is the range of $h(x)$? _____

Is $h(x)$ continuous on its domain? _____

4. <u>Without cutting or pasting</u>, graph the piecewise-defined function shown below by hand. Then, answer the questions that follow.

$$j(x) = \begin{cases} \sqrt[3]{x} & if -8 \le x \le -1 \\ x^3 & if -1 < x < 1 \\ \sqrt[3]{x} & if \quad 1 \le x \le 8 \end{cases}$$



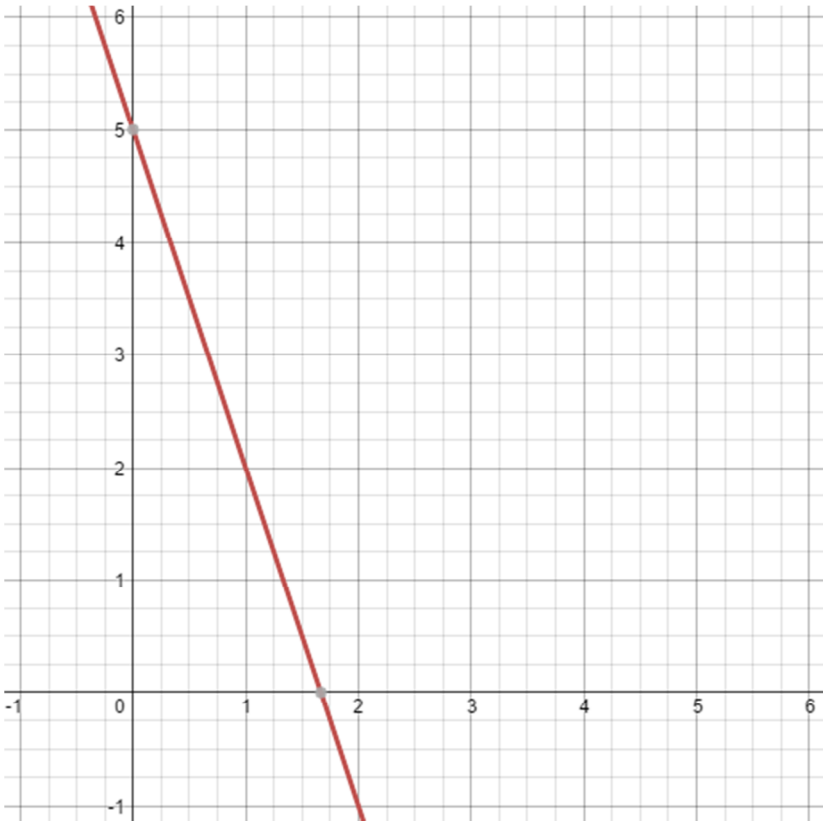
Using interval notation, what is the domain of $j(x)$? _____

Using interval notation, what is the range of $j(x)$? _____

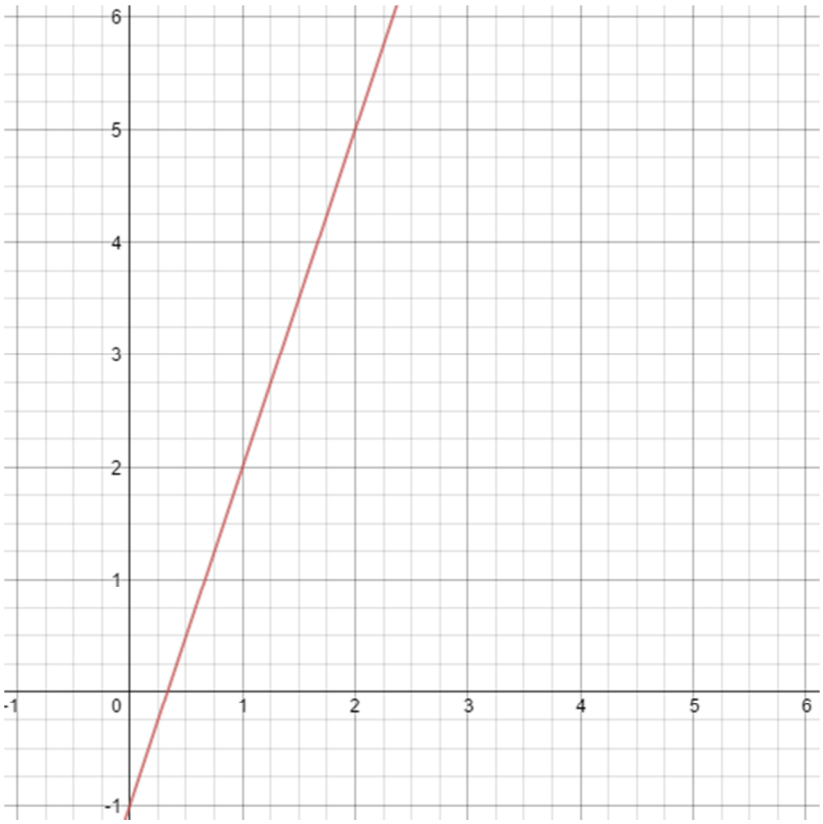
Is $j(x)$ continuous on its domain? _____

Graphs to cut out and use when completing this mini-project appear below. Do not include these pages when you submit your project.

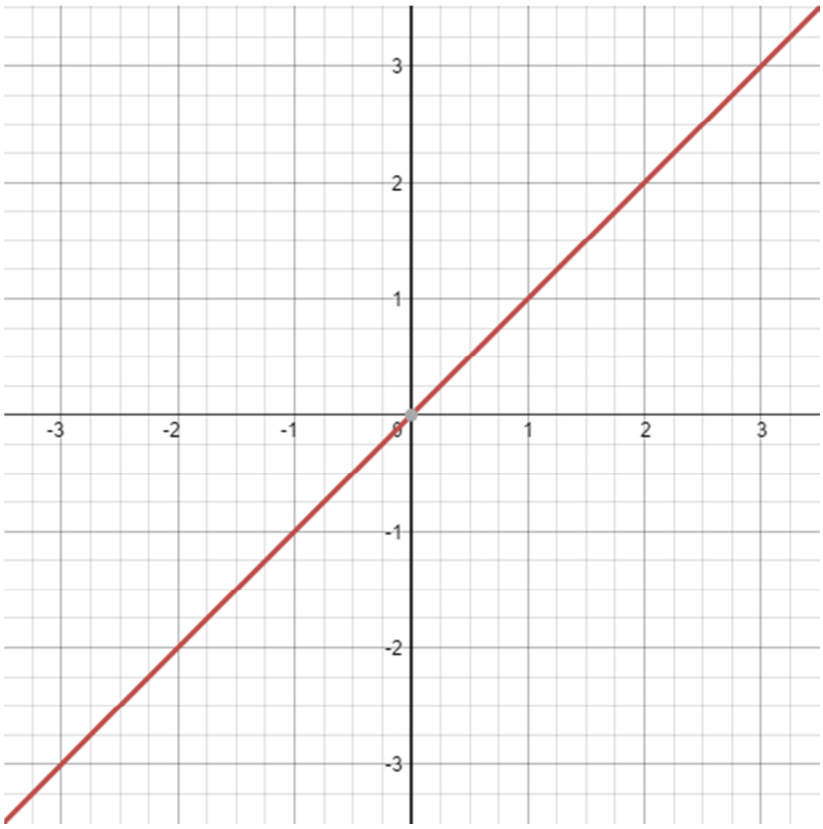
$f(x) = -3x + 5$



$f(x) = 3x - 1$

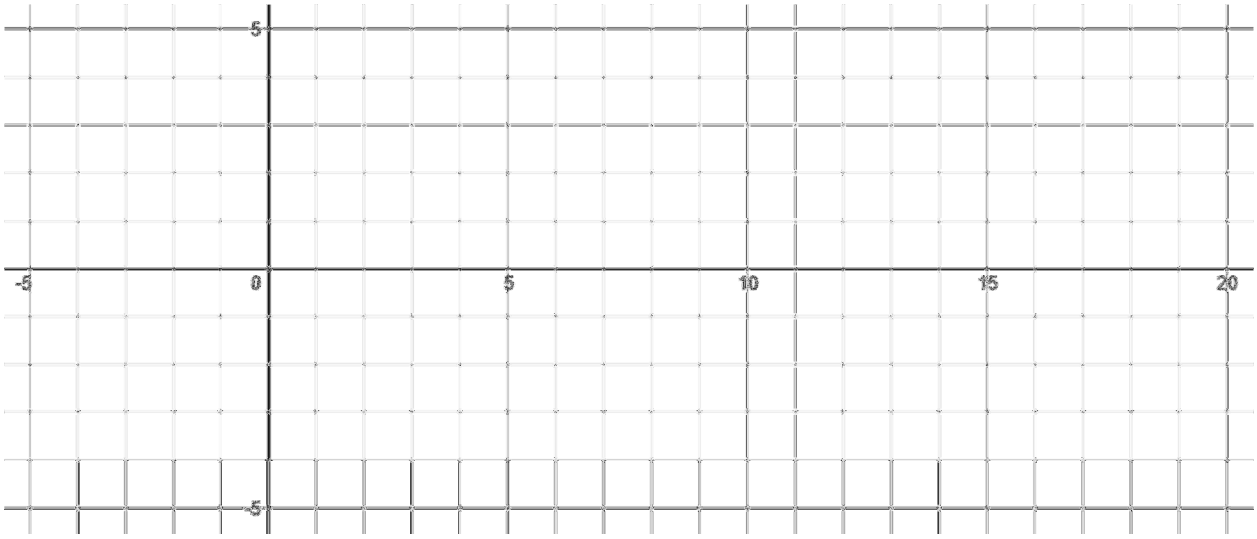


$g(x) = x$



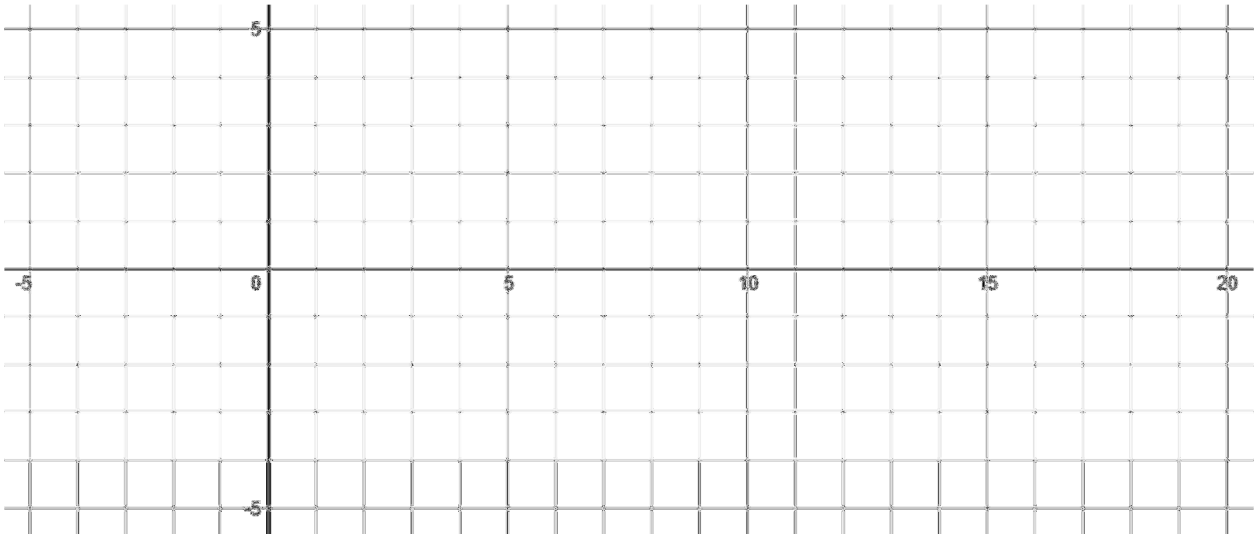
$h(x) = 3$

(You draw the graph)



$h(x) = |x|$

(You draw the graph)



$h(x) = \sqrt{x}$

(You draw the graph)